Extending Layerwise Relevance Propagation using Semiring Annotations

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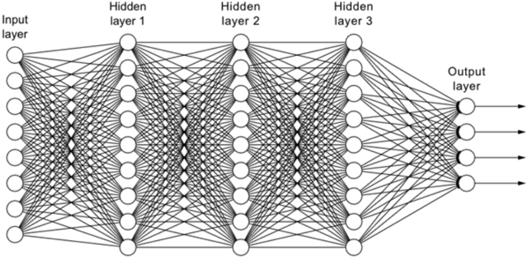
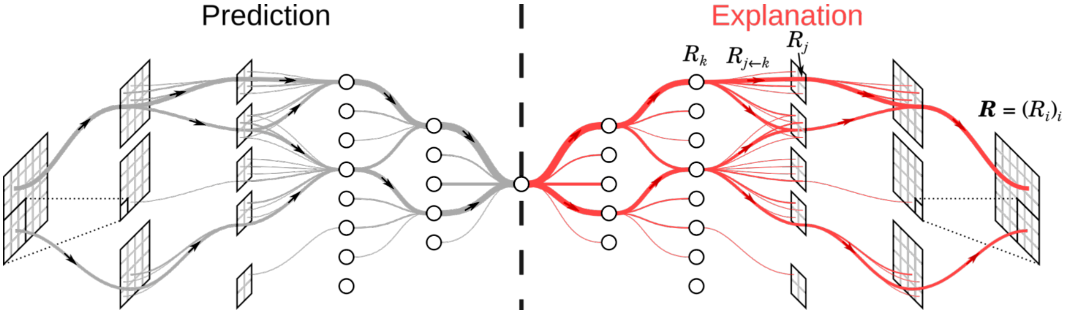
Plan

Problem statement



"castle"

Layerwise Relevance Propagation [11]



Layerwise Relevance Propagation

Initialization

Initialization:

$$R_i^{(L)} = \begin{cases} a_i^{(L)} & \text{if } i = y \text{ (the class we want)} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

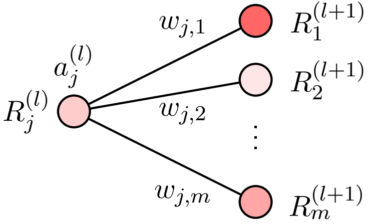
$$\begin{bmatrix} 0 \\ 0 \\ \vdots \\ \mathbf{4.2} \\ \vdots \\ 0 \end{bmatrix} \begin{array}{l} \longrightarrow \texttt{"goldfish"} \\ \longrightarrow \texttt{"street sign"} \\ \\ \longrightarrow \texttt{"castle"} \\ \\ \longrightarrow \texttt{"printer"} \end{array}$$

Layerwise Relevance Propagation

Propagation

LRP-0 rule:

$$R_j^{(l)} = \sum_k \frac{a_j^{(l)} w_{j,k}}{\sum_{j'} a_{j'}^{(l)} w_{j',k}} \cdot R_k^{(l+1)} \tag{LRP-0}$$



Other rules exist (LRP-ε, LRP-γ, $z^{\mathcal{B}}$)

LRP Results visualization

Multilayer Perceptron on MNIST dataset

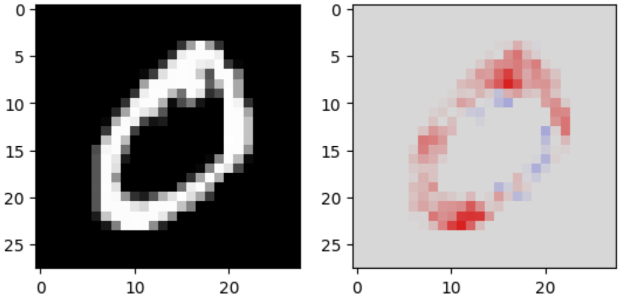


Figure 1: Reference image and relevance for the class 0.

LRP Results visualization

VGG-16 on ImageNet dataset

Figure 2: Reference image

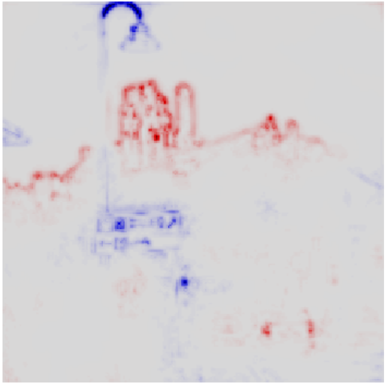


Figure 3: Relevance for the class "castle"

Semiring-based provenance annotations [7, 12]

Definition (Semiring)

A semiring $(\mathbb{K}, \oplus, \otimes, \mathbb{0}, \mathbb{1})$ is such that:

- $\otimes$ distributes over $\oplus$,
- $(\mathbb{K}, \oplus, \mathbb{0})$ is a commutative monoid,
- $(\mathbb{K}, \otimes, \mathbb{1})$ is a monoid such that $\mathbb{0}$ is absorbing

Example

The following structures are semirings:

- Real semiring: $(\mathbb{R}, +, \times, 0, 1)$
- Boolean semiring: $(\{\bot, \top\}, \vee, \wedge, \bot, \top)$
- Counting semiring: $(\mathbb{N}, +, \times, 0, 1)$
- Viterbi semiring: $([0,1], \max, \times, 0, 1)$

Plan

Semiring generalization of the LRP rule

Consider a semiring $(\mathbb{K}, \oplus, \otimes, \mathbb{0}, \mathbb{1})$
Conversion function:

$$\Theta : \mathbb{R} \longrightarrow \mathbb{K}$$

Initialization:

$$R_i^{(L)} = \begin{cases} \mathbb{1} & \text{if } i = y \\ \mathbb{0} & \text{otherwise} \end{cases} \tag{2}$$

Propagation rule:

$$R_j^{(l)} = \bigoplus_k \Theta\left(\frac{a_j^{(l)} w_{j,k}}{\sum_{j'} a_{j'}^{(l)} w_{j',k}} \right) \otimes R_k^{(l+1)} \tag{$\mathbb{K}$-LRP}$$

Semiring generalization of the LRP rule

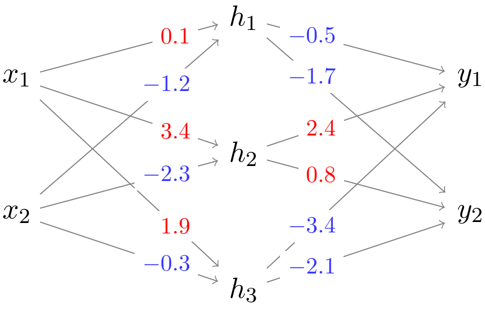


Figure 4: Original network

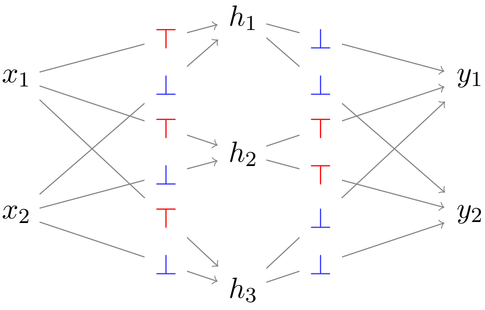
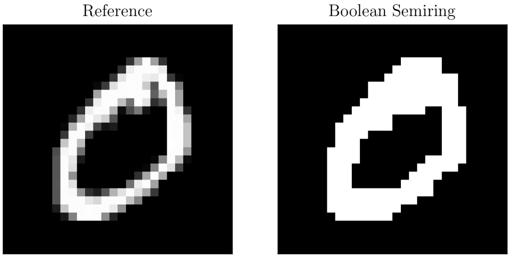


Figure 5: Annotated network (Boolean)

Boolean Semiring
$(\{\bot, \top\}, \vee, \wedge, \bot, \top)$

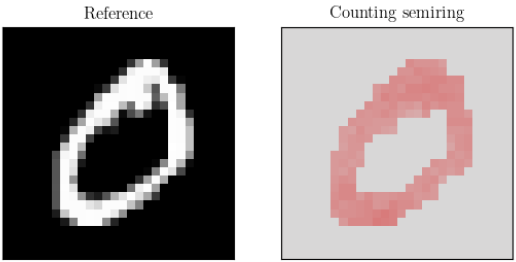
$$\Theta = x \longmapsto \begin{cases} \top & \text{if } x \geqslant \theta \\ \bot & \text{otherwise} \end{cases}$$



Counting Semiring

$(\mathbb{N}, +, \times, 0, 1)$

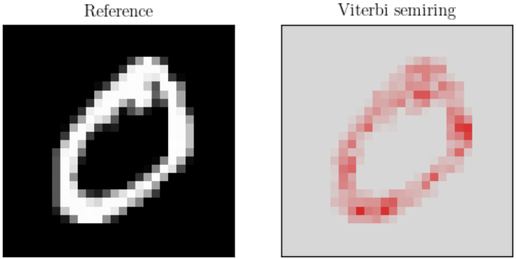
$$\Theta = x \longmapsto \begin{cases} 1 & \text{if } x \geqslant \theta \\ 0 & \text{otherwise} \end{cases}$$



Viterbi Semiring

$$([0,1], \max, \times, 0, 1)$$

$$R_j^{(l)} = \max_k \underbrace{\left(\frac{\left| a_j^{(l)} w_{j,k}^{(l)} \right|}{\max_{j'} \left| a_{j'}^{(l)} w_{j',k}^{(l)} \right|} \right)}_{\in[0,1]} \cdot R_k^{(l+1)}$$



Plan

VGG-16 network

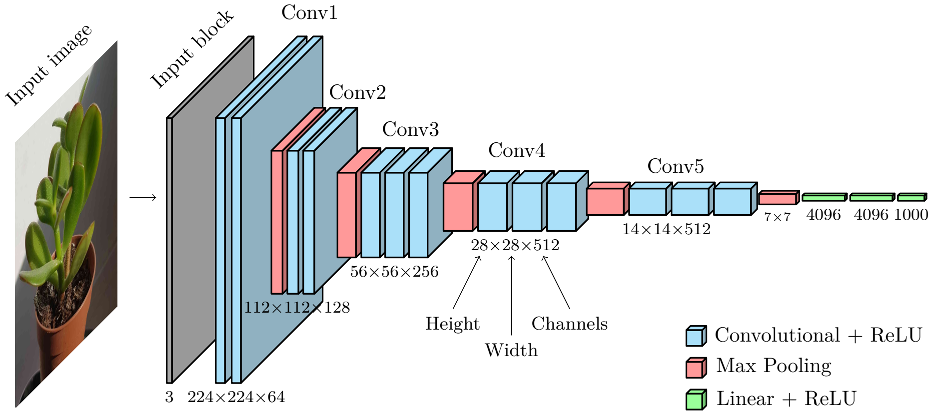
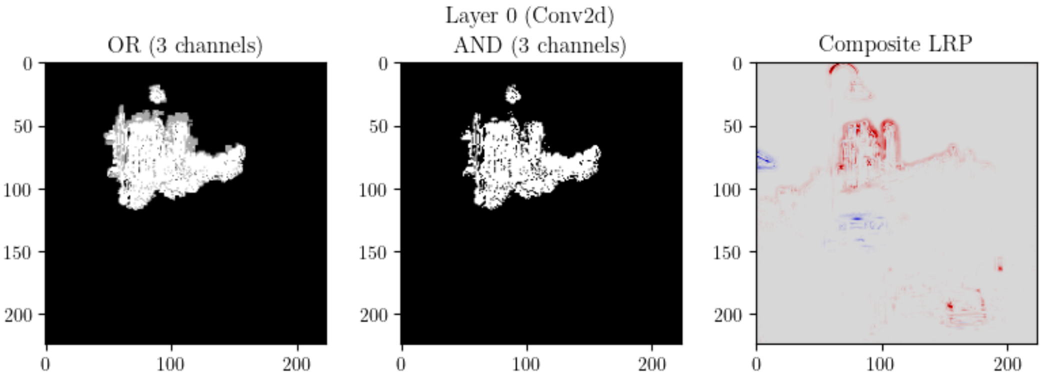


Figure 6: Architecture of the VGG-16 network.

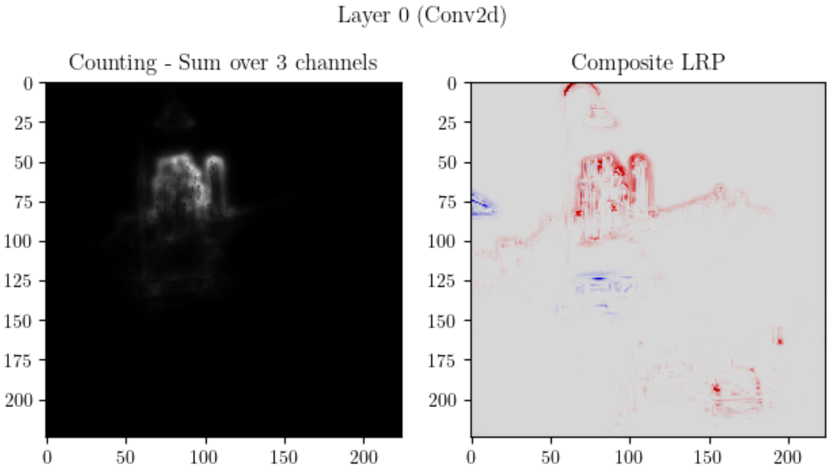
Computing relevance for convolutional layers

$$R_j^{(l)} = \overbrace{\bigoplus_k \underbrace{\Theta\left(\frac{a_j^{(l)} w_{j,k}}{\sum_{j'} a_{j'}^{(l)} w_{j',k}}\right)}_{\text{Convolution over layer } l} \otimes R_k^{(l+1)}}^{\text{Convolution over layer } l+1} \qquad (\mathbb{K}\text{-LRP})$$

Results for VGG-16: Boolean semiring



Results for VGG-16: Counting semiring



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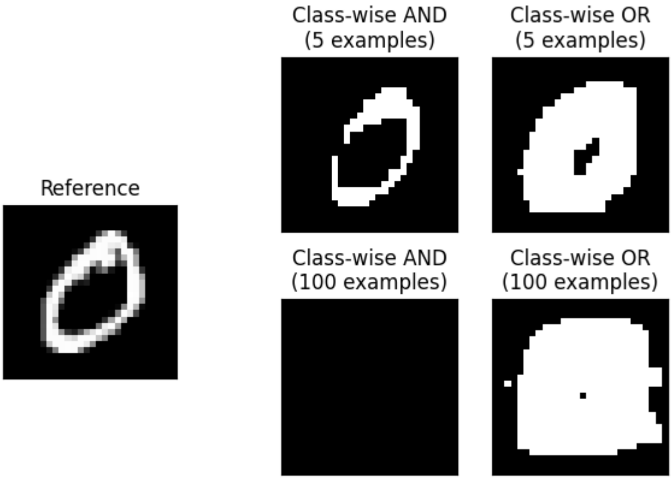
Class-wise mask – Boolean semiring



Network pruning using LRP ranking

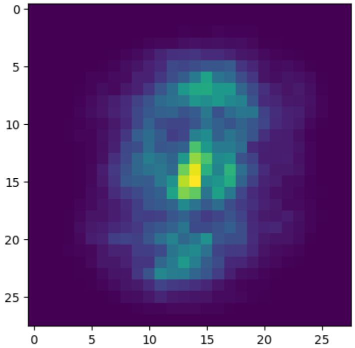
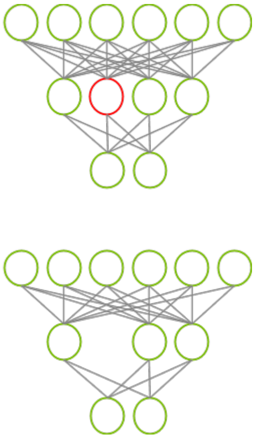
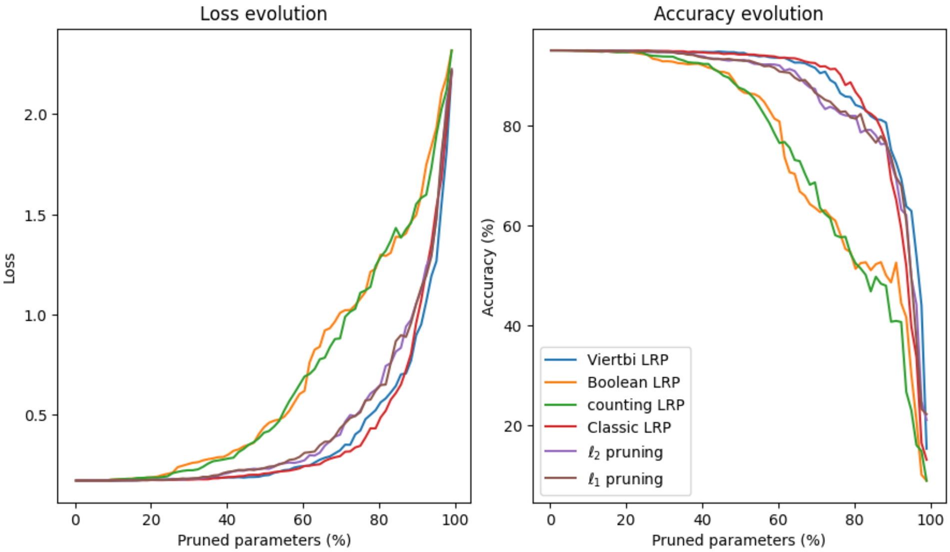


Figure 7: Relevance mean over the training dataset (Input layer)




Loss evolution
Loss
Pruned parameters (%)
0.5
1.0
1.5
2.0
0
20
40
60
80
100
Accuracy evolution
Accuracy (%)
Pruned parameters (%)
20
40
60
80
Viertbi LRP
Boolean LRP
counting LRP
Classic LRP
ℓ_2 pruning
ℓ_1 pruning

Conclusion

- We extended the Layerwise Relevance Propagation method to semirings
- We applied this method to the MNIST dataset and the VGG-16 network
- We showed that the method can be used for image mask computation and network pruning

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